Introduction to Mathematical Epidemiology

Revision notes of the lecture by G. A. Funk, 17 November 2011

Q1: Please give an *intuitive* (i.e. non-mathematical) description of the following epidemiological characteristics:

- *force of infection* (denoted $\boldsymbol{\lambda}$);
- *basic reproductive ratio* (denoted $\mathbf{R_0}$);
- *herd immunity threshold* (denoted **H** or $\mathbf{p_c}$).

Q2: What is/are the main difference(s) between *epidemiological* and *endemic* mathematical models?

Q3: Can you derive either the *force of infection* ($\boldsymbol{\lambda}$) or the *basic reproductive ratio* ($\mathbf{R_0}$) based on a very simple *epidemiological* model? Which assumptions do you make? Explain why they are sensible.

Q4: Can you, based on the *structure* of the model equations, say something about the relative proportions of the host population in the different classes (S, I and R)? What consequences can this have for the *abundance* of the host population (e.g. plague in medieval Europe or rinderpest in Africa ~1890)?

Q5: Suppose two infectious diseases A and B. The $\mathbf{R_0}$ of disease A is 5, the $\mathbf{R_0}$ of disese B is 20. Imagine that you managed to develop a vaccine for each of the diseases with a respective *efficacy* of 90% (i.e. the vaccine protects 90% of the vaccinated individuals but fails to protect 10% of them). Is it possible to control disease A or disease B with the vaccines you developed?

control of disease A (R_0=5)	yes ❑	no ❑
control of disease B (R_0=20)	yes ❑	no ❑

Should you have ticked no, how high has the efficacy to be *at least* in order to achieve control over the respective disease?

For discussion: What kind of *practical problems* would you face/expect?

Q6: Which infectious disesase does interest you most? Search the very first scientific publication that estimated its $\mathbf{R_0}$. How was it done? Can you follow it? Ask three GPs (medical doctors) approximately when in life one normally contracts the disease. What does this information tell you regarding the value of $\mathbf{R_0}$ (close to one or much larger)?

Q7: Go to the website of the Swiss Federal Office of Public Health (BAG) or the CDC (USA) and search either for case notification or sero-epidemiological data of an infectious disease. Can you re-do the calculations shown in the handout that will ultimately lead to $\boldsymbol{\lambda}$, $\mathbf{R_0}$ and **H**? If not: Where did you stuck? Why?

Example: Visit the website *www.bag.admin.ch* of the Swiss Federal Office of Public Health (BAG), then go to *Themen -> Meldesysteme -> Meldepflichtige Infektionskrankheiten -> Wöchentliche Fallzahlen -> Datendetails* and look for a data sheet providing age stratified case notification data.

Q8: In which modern fields – other than infectious diseases – are concepts from epidemiology applied with great success? Hints: Spread of information; human behaviour and 'herding'.

Q9: What is the difference between *epidemics* and *syndemics*? How does *cross-immunity* fit with the concept of syndemics? Any example?

Glossary:
(Index-)Case, Outbreak, Epidemic, Endemic, Pandemic, Syndemic

Please write/sketch your answers on separate sheets, discuss them with your colleagues, and don't hesitate to ask the course instructors for advice or help.

Good luck!
G. A. Funk

Summary of models & formulae:

Basic *epidemiological* model

$$\begin{aligned} dS/dt &= -\beta IS \\ dI/dt &= \beta IS - \mu I = \lambda S - \mu I \\ dR/dt &= \mu I \end{aligned}$$

Basic *endemic* model (with demography and pathogen induced mortality)

$$\begin{aligned} dS/dt &= b - \delta S - \beta IS \\ dI/dt &= \beta IS - \mu I - (\delta + \nu)I \\ dR/dt &= \mu I - \delta R \end{aligned}$$

Force of infection, λ (Anderson & May, 1983 and 1985)

$$\lambda_i = -\ln[(1-p_{i+1}) / (1-p_i)]$$

note: λ_i's are age specific over the intervals i, i+1

Basic reproductive ratio, R_0 (Anderson & May, 1991)

$$R_0 = (\lambda \cdot L) / (1 - \mathrm{Exp}[-\lambda \cdot L])$$

Herd immunity threshld, H or p_c (Anderson & May, 1991)

100% vaccine efficacy (ε=1)

$$H = 1 - 1/R_0$$

less than 100% vaccine efficacy ($0<\varepsilon<1$)

$$H = [1 - 1/R_0] / \varepsilon$$