II.

3) a) $p_0 = \frac{2}{6}$

$p_i = \frac{1}{6} \qquad \forall\ 1 \le i \le 4$ 1

b) $E[X] = \sum_{i=1}^{4} i \cdot \frac{1}{6} = \frac{1}{6} \cdot \frac{5 \cdot 4}{2} = \frac{10}{6} = \frac{5}{3}$ 1

c) $V[X] = \sum_{i=0}^{5} \left(i - \frac{5}{3}\right)^2 p_i = \dots$ oder 1

$= \sum_{i=1}^{4} \left(1 - \frac{5}{3}\right)^2 \cdot \frac{1}{6} + \left(0 - \frac{5}{3}\right)^2 \cdot \frac{2}{6} = 2.22$

$E[X^2] = \sum_{i=1}^{4} i^2 \cdot \frac{1}{6} = \frac{1}{6}\{1+4+9+16\} = 5$

$V[X] = E[X^2] - E[X]^2 = 5 - \left(\frac{5}{3}\right)^2 = 2.22 = \frac{20}{9}$

d) ✓ $\left(\frac{1}{2}\right)$ 1

$E\left[\frac{1}{1+X^2}\right] = \sum_{i=0}^{4} \frac{1}{1+i^2} p_i = \frac{2}{6} + \frac{1}{2}\frac{1}{6} + \frac{1}{5}\frac{1}{6} + \frac{1}{10}\frac{1}{6} + \frac{1}{17}\frac{1}{6}$

$= \frac{2}{6} + \frac{1}{6}\{0.859\} = 0.476 = \frac{81}{170}$

e) ist $E[X] = \frac{5}{3}$; [illegible] 1

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4) a) Jensen: e ist konvex; $E[e^X] \ge e^{E[X]} = e^0 = 1$ 2

b) $E[e^X] = E[e^X 1_{\{X \le 2\sigma\}}(X) + e^X 1_{\{X > 2\sigma\}}(X)]$

$\ge 0 + E[e^X 1_{\{X > 2\sigma\}}(X)]$

$\ge E[e^{2\sigma} 1_{\{X > 2\sigma\}}(X)] = e^{2\sigma} E[1_{\{X > 2\sigma\}}(X)]$

$= e^{2\sigma} P[X > 2\sigma] \ge e^{2\sigma} \cdot 0.022 \xrightarrow[\sigma \to \infty]{} \infty$

q.e.d.

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