Crash Course in Statistics for Neuroscience Center Zurich University of Zurich

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Summary Chapter 1-5

Conditional Probability

$$P[A|B] := \frac{P[A \cap B]}{P[B]},$$

leading to

$$P[A|B]P[B] = P[A \cap B] = P[B|A]P[A].$$

Independent Events

$$P[A \cap B] = P[A]P[B]$$

or better:

$$P[A|B] = P[A]$$

(or $P[B|A] = P[B]$ respectively).

Cumulative Distribution Function F

$$F(a) := P[X \leq a] := P[\{\omega | X(\omega) \leq a\}].$$

Often written F_X instead of F for better identification.

continuous Random Variables ... if F_Y can be written as an integral in the form

$$F_Y(a) := P[Y \leq a] = \int_{-\infty}^{a} f(u)du,$$

with f a nonnegative function on $\mathbb{R}$ such that

$$\int_{-\infty}^{\infty} f(u)du = 1.$$

We call f density or density function of Y (sometimes denoted as f_Y for better identification). En bref: discrete is isolated points and continuous is on entire intervals.

Continuous Random Variables satisfy (see Normal Distribution as an illustration):

1. For $a < b$ we have:

$$P[a < X \leq b] = P[X \leq b] - P[X \leq a] = F(b) - F(a) = \int_a^b f(x)dx.$$

2. $P[X = x_0] = \int_{x_0}^{x_0} f(x)dx = 0.$

3. Because of remark 2 we have for $a < b$:

$$\begin{aligned} P[a \leq X \leq b] &= P[a < X \leq b] = P[a < X < b] = P[a \leq X < b] \\ &= P[X \leq b] - P[X \leq a] = F(b) - F(a) = \int_a^b f(x)dx. \end{aligned}$$

4. If F is differentiable:

$$F'(x) = f(x).$$

Expectation (mean)

$$E[X] := \sum_x xP[X = x].$$

or

$$E[Y] := \int_{-\infty}^{\infty} uf_Y(u)du.$$

$f_Y(u)$ is the density function.

Variance

$$V[X] := E[(X - \mu_X)^2] = \sum_x (x - \mu_X)^2 P[X = x].$$

or

$$V[Y] := E[(Y - \mu_Y)^2] = \int_{-\infty}^{\infty} (y - \mu_Y)^2 f_Y(y)dy$$

where $f_Y(y)$ is the density function of Y. Standard deviation is (for both, discrete and continuous):

$$sd[X] := \sqrt{V[X]}.$$

Rules:

$E[aX] = aE[X]$ [in German: "Konstante herausziehen"]

$E[X+Y] = E[X] + E[Y]$ [in German: "Summen auseinanderziehen"]

$V[aX+b] = a^2 V[X]$ [in German: "Konstante quadratisch raus"]

Let $(X_i)_{i=1}^n$ be a sequence of independent random variables. Then:

$$V[\sum_{i=1}^n X_i] = \sum_{i=1}^n V[X_i]$$

[in German: "Varianz der Summe ist Summe der Varianzen"].

Let $X_1, \ldots, X_n$ be a sequence of iid RV's,

$$\bar{X} := \frac{1}{n}\sum_{k=1}^n X_k$$

and let $\sigma^2 := V[X_1]$. $V[\bar{X}] = \frac{\sigma^2}{n}$ and $sd[\bar{X}] = \frac{\sigma}{\sqrt{n}}$.

Binomial Bin(n,p)

X_i, $1 \le i \le n$, n iid Be(p)-R.V. $Y := \sum_{i=1}^n X_i$. Y so called Binomial RV with parameters n and p; Bin(n,p). $E[Y] = np$ and $V[Y] = np(1-p)$. Possible values: natural numbers $0 \le y \le n$:

$$P[Y = y] = \binom{n}{y} p^y (1-p)^{n-y}.$$

"How many successes in n trials?"

Normal $\mathbf{N}(\mu, \sigma^2)$

Very important. Density:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad x \in \mathbb{R},$$

$E[X] = \mu$ and $V[X] = \sigma^2$.

N1 Let X be $\mathcal{N}(\mu_1, \sigma_1^2)$ and Y be $\mathcal{N}(\mu_2, \sigma_2^2)$, X independent of Y. Then $X + Y$ has a

$$\mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$$

distribution.

N2 "Z-Transform": If X has a $\mathcal{N}(\mu, \sigma^2)$ distribution, then

$$\frac{X - \mu}{\sigma}$$

has a $\mathcal{N}(0, 1)$-distribution (which can be found in tables in books and libraries of statistical packages). $\mathcal{N}(0, 1)$ is called "standard" normal.

N3 approximate results: for any σ: let X be $\mathcal{N}(0, \sigma^2)$ RV, then

$$P[|X| > \sigma] \doteq 33\%$$

and

$$P[|X| > 2\sigma] \doteq 5\%.$$

Generally: Normal distribution has 66 % of it's probability within 1 standard deviation away from the mean and even 95 % within 2 standard deviations away from the mean:

What I want you to memorize from chapter 5:

Due to LLN arithmetic mean converges to (theoretical, true) mean (under very weak mathematical assumptions)